

The ratio of decay widths of $X(3872)$ to $\psi'\gamma$ and $J/\psi\gamma$ as a test of the $X(3872)$ dynamical structure

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Radiative decays of $X(3872)$ with $J^{PC} = 1^{++}$ are studied in the coupled-channel approach, where the $c\bar{c}$ states are described by relativistic string Hamiltonian, while for the decay channels DD^* a string breaking mechanism is used. Within this method a sharp peak and correct mass shift of the 2^3P_1 charmonium state just to the D^0D^{*0} threshold was already obtained for a prescribed channel coupling to the DD^* decay channels. For the same value of coupling the normalized wave function (w.f.) of $X(3872)$ acquires admixture of the 1^3P_1 component with the weight $c_1 = 0.153$ ($\theta = 8.8^\circ$), which increases the transition rate $\Gamma(X(3872) \rightarrow J/\psi\gamma)$ up to 50-70 keV, making the ratio $R = \frac{\mathcal{B}(X(3872) \rightarrow \psi'\gamma)}{\mathcal{B}(X(3872) \rightarrow J/\psi\gamma)} = 0.8 \pm 0.2$ (*th*) significantly smaller, as compared to $R \simeq 5$ for $X(3872)$ as a purely 2^3P_1 state.

I. INTRODUCTION

The $X(3872)$ was discovered by Belle as a narrow peak in $J/\psi\pi\pi\pi$ invariant mass distribution in decays $B \rightarrow J/\psi\pi\pi K$ [1] and later confirmed by the CDF, D0, and BaBar Collaborations [2]. It has several exotic properties, very small width $\Gamma < 2.3$ MeV and the mass very close to the D^0D^{*0} threshold [3],[4]. The even charge parity $C = +$ of $X(3872)$

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is now well established [5], while two most plausible assignments for its quantum numbers, $J^{PC} = 1^{++}$ and 2^{-+} , are still discussed [6], [7].

To understand the nature of $X(3872)$ a special role belongs to radiative decays, $X(3872) \rightarrow J/\psi\gamma$ and $X(3872) \rightarrow \psi'\gamma$. The first evidence for the decay $X(3872) \rightarrow J/\psi\gamma$ was obtained by Belle [8] and confirmed by BaBar [9]; later the BaBar has observed radiative decay $X(3872) \rightarrow \psi'\gamma$ with the branching fraction ratio $R = \frac{\mathcal{B}(X(3872) \rightarrow \psi'\gamma)}{\mathcal{B}(X(3872) \rightarrow J/\psi\gamma)} = 3.4 \pm 1.4$ [10]. Knowledge of this ratio is of special importance for theory, because the rates of these radiative decays vary widely in different theoretical models [11]-[15]. If $X(3872)$ is considered as a conventional 2^3P_1 charmonium state, then the characteristic value of R is rather large, $R \simeq 4 - 6$ [12]- [14], being in general in agreement with the BaBar number 3.4 ± 1.4 . In molecular picture the radiative decay $X(3872) \rightarrow \psi'\gamma$ is suppressed and the ratio R should be much smaller [11].

However, in 2010 on a larger sample of decays $B \rightarrow X(3872)K$ the Belle has not found evidence for the radiative decay $X(3872) \rightarrow \psi'\gamma$, giving the upper limit $R < 2.1$ [16]. This number does not agree with the representation of $X(3872)$ as a purely 2^3P_1 charmonium state.

Existing experimental uncertainty calls for new studies of coupled-channel (CC) effects for $X(3872)$. In [13] the authors have anticipated “... a significant DD^* component in $X(3872)$, even if it is dominantly a $c\bar{c}$ state”. For that the 3P_0 model was used in [13] and the Cornell many-channel model was considered in [14]. However, in spite of these many-channel calculations there predicted values of R have appeared to be close to those obtained for $X(3872)$ as a purely 2^3P_1 charmonium state: $R = 5.8$ in [13] and $R = 5.0$ in [14].

Here we consider $X(3872)$ with $J^{PC} = 1^{++}$ in the CC approach, where a coupling to the DD^* channels is defined by the relativistic string-breaking mechanism, which was already applied to $X(3872)$ in [17], [18], explaining it as 2^3P_1 charmonium state shifted down and appearing as a sharp peak just at the D^0D^{*0} threshold. Besides, the scattering amplitude and a production cross section were calculated there, being in qualitative agreement with experiment. Here we apply this method for calculations of the radiative decay rates for $X(3872)$ and show that due to the same CC mechanism (with the coupling of the same strength) an admixture of the 1^3P_1 component to the $X(3872)$ w.f. appears to be not large, $\sim 15\%$; nevertheless, this component strongly affects the value of the partial width $\Gamma_1 = \Gamma(X(3872) \rightarrow J/\psi\gamma)$ and decreases the ratio R .

II. COUPLED-CHANNEL MECHANISM

We use here the string decay Lagrangian of the 3P_0 type for the decay $c\bar{c} \rightarrow (c\bar{q})(\bar{c}q)$ [18]:

$$\mathcal{L}_{sd} = \int \bar{\psi}_q M_\omega \psi_q d^4x \quad (1)$$

where the light quark bispinors are treated in the limit of large m_c , which allows us to go over to the reduced (2×2) form of the decay matrix elements (m.e.). Also to simplify calculations the actual w.f. of $c\bar{c}$ states, calculated in [19] with the use of the relativistic string Hamiltonian (RSH), is fitted here by five (or three) oscillator w.f. (SHO), while the D meson w.f. is described by a single SHO term with few percent accuracy with the parameter $\beta \simeq 0.48$. In this case the factor M_ω in (1) is $M_\omega \simeq \frac{2\sigma}{\beta} \simeq 0.8$ GeV, which produces correct total width of $\psi(3770)$ and it will be used below.

The transition m.e. for the decays $(c\bar{c})_n \rightarrow (D\bar{D}), (DD^*), (D^*D^*)$ are denoted here as $n \rightarrow n_2, n_3$, and in the 2×2 formalism this m.e. reduces to

$$J_{nn_2n_3}(\mathbf{p}) = \frac{\gamma}{\sqrt{N_c}} \int y_{123} \frac{d^3\mathbf{q}}{(2\pi)^3} \Psi_n^+(\mathbf{p} + \mathbf{q}) \psi_{n_2}(\mathbf{q}) \psi_{n_3}(\mathbf{q}). \quad (2)$$

Here $\gamma = \frac{2M_\omega}{\langle m_q + U - V_D + \epsilon_0 \rangle}$, where average of the Dirac denominator (with scalar confining potential $U = \sigma r$ and vector potential $V_D = -\frac{4\alpha}{3r}$) is calculated and yields $\gamma = 1.4$. The factor \bar{y}_{123} contains a trace of spin-angular variables (for details see [18], [20]).

The intermediate decay channel, like DD^* , induces an additional interaction “potential” $V_{CC}(\vec{q}, \vec{q}', E)$ (here the quotation marks imply nonlocality and energy dependence of this potential):

$$V_{CC}(\mathbf{q}, \mathbf{q}', E) = \sum_{n_2n_3} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{X_{n_2n_3}(\mathbf{q}, \mathbf{p}) X_{n_2n_3}^+(\mathbf{q}', \mathbf{p})}{E - E_{n_2n_3}(\mathbf{p})}, \quad (3)$$

where

$$X_{n_2n_3}(\mathbf{q}, \mathbf{p}) = \frac{\gamma}{\sqrt{N_c}} \bar{y}_{123}(\mathbf{q}, \mathbf{p}) \psi_{n_2}(\mathbf{q} - \mathbf{p}) \psi_{n_3}(\mathbf{q} - \mathbf{p}). \quad (4)$$

Using (3) and (4) one can find how the energy eigenvalues (e.v.) and the w.f. of a state $(c\bar{c})_n$ change due to the interaction V_{CC} . In particular, in the first order of perturbation theory one has

$$E_n^{(1)} = E_n + w_{nn}(E_n), \quad (5)$$

$$\psi_n^{(1)} = \psi_n + \sum_{m \neq n} \frac{w_{nm}(E_n)}{E_n - E_m} \psi_m, \quad (6)$$

where ψ_n , E_n refer to the unperturbed $(c\bar{c})_n$ system and the m.e. $w_{nm}(E)$ is

$$\begin{aligned} w_{nm}(E) &= \int \psi_n(\mathbf{q}) V_{CC}(\mathbf{q}, \mathbf{q}', E) \psi_m(\mathbf{q}') \frac{d^3 \mathbf{q}}{(2\pi)^3} \frac{d^3 \mathbf{q}'}{(2\pi)^3} = \\ &= \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \sum_{n_2 n_3} \frac{J_{nn_2 n_3}(\mathbf{p}) J_{mn_2 n_3}^+(\mathbf{p})}{E - E_{n_2 m_3}(\mathbf{p})}. \end{aligned} \quad (7)$$

Notice, that if the CC interaction is strong, then one should take into account this interaction to all orders, summing the infinite series over V_{CC} (or w_{nm}). As a result, one obtains the full Green's function for an arbitrary $Q\bar{Q}$ system (our result formally coincide with those from [21], although differences occur in concrete expressions for w_{nm} , because our interaction and decay mechanism differ from those in [21]):

$$G_{Q\bar{Q}}(1, 2; E) = \sum_{n, m} \Psi_{Q\bar{Q}}^{(n)}(1) (\hat{E} - E + \hat{w})_{nm}^{-1} \Psi_{Q\bar{Q}}^{+(m)}(2), \quad (8)$$

here $\hat{E}_{nm} = E_n \delta_{nm}$. Then the energy e.v. are to be found from the zeros of the determinant

$$\det(E - \hat{E} - \hat{w}) = 0. \quad (9)$$

If the mixing of the state n with other states, $m \neq n$, is neglected, one obtains a nonlinear equation for the e.v. E_n^* with a correction:

$$E_n^{(*)} = E_n + w_{nn}(E_n^{(*)}). \quad (10)$$

Note that E_n^* can be a complex number and occur on the second sheet in the complex plane with a cut from the threshold $n_2 n_3$ to infinity. The expression (10) was used in [17] to find the position of the pole for shifted $2^3 P_1$ charmonium state, while its unperturbed value E_2 was calculated with the use of RSH [22] to be $E_2 = 3948 \pm 10$ MeV. In [17] the decays have included $D_0 D_0^*$ and $D^+ D^-$ channels and γ was used as a free parameter. Then the position of the resulting pole (10) and the production cross section have been calculated, giving the pole position exactly at the $D_0 D_0^*$ threshold for $\gamma = 1.2$, which is close to expected

number $\gamma = \frac{M_\omega}{\langle m_q + u - V_D + \varepsilon_0 \rangle} = 1.4$ for $M_\omega = 0.8$ GeV. (From Fig.2 in [17] one can see that for this value of γ the production curve agrees well qualitatively with experiment.)

Knowing the mass shift of $X(3872)$, one has also to find next order corrections to the w.f., thus going beyond no-mixing approximation. At first, we use perturbation theory, when admixture of the states $m^3P_1 \equiv m1^{++}$ to the 2^3P_1 state is given by the (6),

$$c_m^{(1)} = \frac{w_{2m}}{E_2 - E_m}, \quad m = 1, 3, 4, \dots \quad (11)$$

Calculations give $w_{21} = 0.085$ GeV, $w_{23} = -0.0008$ GeV, while $E_2 - E_1 = 0.425$ GeV, so that main contribution comes from the $m = 1$ (1^3P_1) state with the mixing parameter

$$c_1^{(1)} = \frac{w_{21}}{E_2 - E_1} \cong 0.20. \quad (12)$$

This correction $c_1^{(1)}$ is not small and calls for a more accurate calculations, beyond perturbation theory.

To this end we first write general expressions for the yield of particles $\gamma, \pi, \rho, \omega$ etc. from the system, originally born as a $(Q\bar{Q})$ system in e^+e^- or B meson decay by an operator \hat{B} . The Green's function for the system can be written as

$$G_{Q\bar{Q}}^{(\mathcal{B}\mathcal{B})} = \sum_{n,m} (\hat{B}\psi_n) \left(\frac{1}{\hat{E} - E + \hat{w}} \right)_{nm} (\hat{B}\psi_m), \quad (13)$$

where e.g. $(\hat{B}\psi_n) \sim \psi_n(0)$ for the e^+e^- production. We now can attribute particle (i) production from Q, \bar{Q} lines adding the corresponding self energy parts to \hat{E} , $(\hat{E})_{nm} = E_n \delta_{nm} + \sum_{nm}^{(i)}(E)$, and the production from light quark lines to $w_{mn}(e)$, $w_{mn}(E) \rightarrow w_{mn}(E) + w_{mn}^{(i)}(E)$. Then the yield for particles i can be written as

$$Y_i(E) = \sum_{n,m,l,q} (\hat{B}\psi_n) \left(\frac{1}{\hat{E} - E + \hat{w}} \right)_{nm} \frac{\Delta^{(i)}}{2i} (\Sigma_{ml}^{(i)} + w_{ml}^{(i)}) \left(\frac{1}{\hat{E} - E + \hat{w}} \right)_{lq}^* (\hat{B}\psi_q)^*. \quad (14)$$

One can define now effective $(Q\bar{Q})$ wave function $\Psi_{Q\bar{Q}}$, which actually participates at the vertex of emission of particles i ,

$$\Psi_{Q\bar{Q}}^{(\mathcal{B})} = \sum_{k,l} \Psi_k \left(\frac{1}{\hat{E} - E + \hat{w}} \right)_{kl} (\hat{B}\psi_l) \equiv \sum_k a_k \Psi_k. \quad (15)$$

Keeping only two eigenfunctions, one has

$$a_1 = \frac{E_2 - E + w_{22}}{\det} (\hat{B}\psi_1) - \frac{w_{21}}{\det} (\hat{B}\psi_2) \quad (16)$$

$$a_2 = \frac{E_1 - E + w_{11}}{\det} (\hat{B}\psi_2) - \frac{w_{12}}{\det} (\hat{B}\psi_1), \quad (17)$$

where $\det \equiv \det(\hat{E} - E + \hat{w})$.

From (21) one obtains the ratio

$$\frac{a_1}{a_2} = \frac{w_{21} + (E_R - E) \frac{(\hat{B}\psi_1)}{(\hat{B}\psi_2)}}{\Delta_1 - w_{11} + w_{12} \frac{(\hat{B}\psi_1)}{(\hat{B}\psi_2)}}, \quad \Delta_1 \equiv E_R - E_1 = 362 \text{ MeV}. \quad (18)$$

Neglecting $\frac{(\hat{B}\psi_1)}{(\hat{B}\psi_2)}$, and for $E = E_R = 3872 \text{ MeV}$ one obtains

$$\frac{a_1}{a_2} = \frac{w_{21}}{\Delta_1 - w_{11}} = 0.179, \quad (19)$$

while approximating in B -decay production $\frac{(\hat{B}\psi_1)}{(\hat{B}\psi_2)} \approx \frac{\psi'_1(0)}{\psi'_2(0)} \approx 0.85$ one obtains

$$\frac{a_1}{a_2} \cong 0.155. \quad (20)$$

One can see in Table I, that the $n = 3 \ ^3P_1$ state gives a negligible admixture. The values of w_{mn} for $E = E_R$ are computed according to Eq. 7 with w.f. obtained in [19] and are given in Table I.

TABLE I: The m.e. w_{nm} (in GeV) between $n \ ^3P_1$ and $m \ ^3P_1$ states for two approximations of exact w.f.

nm	11	12	22	32
(w_{nm})	-0.320	0.122	- 0.099	-0.0003
$5SHO$				
w_{nm}	-0.319	0.121	- 0.098	-0.0011
$3SHO$				

Then using (20) the w.f. of $X(3872)$ can be presented with a good accuracy as

$$\varphi(X(3872)) = 0.988 \ \varphi(2 \ ^3P_1) + 0.153 \ \varphi(1 \ ^3P_1) \quad (21)$$

III. RADIATIVE DECAYS

Electric dipole transitions between an initial state (i) $n \ ^3P_1$ state and a final (f) state $m \ ^3S_1$ are defined by the partial width [7], [24],

$$\Gamma(i \xrightarrow{E1} \gamma + f) = \frac{4}{3} \alpha e_Q^2 E_\gamma^3 (2J_f + 1) S_{if}^E |\mathcal{E}_{if}|^2, \quad (22)$$

where the statistical factor $S_{if}^E = S_{fi}^E$ is

$$S_{if}^E = \max(l, l') \left\{ \begin{matrix} J & 1 & J' \\ l' & s & l \end{matrix} \right\}^2. \quad (23)$$

For the transitions between the n^3P_J and m^3S_1 (m^3D_1) states with the same spin $S = 1$, the coefficient $S_{if}^E = \frac{1}{9}(\frac{1}{18})$.

To calculate m.e. \mathcal{E}_{if} we use RSH H_0 [22], which is simplified in case of heavy quarkonia when one can neglect a string and self-energy corrections, arriving at a simple form (widely used in relativistic potential models with the constituent quark masses in the kinetic term [25],[26]):

$$H_0 = 2\sqrt{\mathbf{p}^2 + m_c^2} + V_B(r). \quad (24)$$

By derivation, in (24) the mass of the c quark cannot be chosen arbitrarily and must be equal to the pole mass of a c quark, $m_c \simeq 1.42$ GeV. The pole mass takes into account perturbative in $\alpha_s(m_c)$ corrections and corresponds to the conventional current mass $\bar{m}_c(\bar{m}_c) = 1.22$ GeV [27] (here $m_c = 1424$ MeV is used).

The potential $V_B(r)$ taken,

$$V_B(r) = \sigma r - \frac{4\alpha_B(r)}{3r}, \quad (25)$$

contains the string tension ($\sigma = 0.18$ GeV²), which cannot be considered as a fitting parameter, because it is fixed by the slope of the Regge trajectories for light mesons. In the vector strong coupling $\alpha_B(r)$ the asymptotic freedom behavior is taken into account with the QCD constant Λ_B , which is defined by $\Lambda_{\overline{MS}}$: $\Lambda_B(n_f = 4) = 1.4238 \Lambda_{\overline{MS}}(n_f = 4) = 370$ MeV, the latter is supposed to be known; in our choice $\Lambda_B(n_f = 4)$ corresponds to $\Lambda_{\overline{MS}}(n_f = 4) = 261$ MeV. At large distances $\alpha_B(r)$ freezes at the value $\alpha_{\text{crit}} = 0.60$.

Then for a given multiplet nl the centroid mass $M_{\text{cog}}(nl)$ is equal to the e.v. of the spinless Salpeter equation (SSE):

$$H_0 \varphi_{nl} = M_0(nl) \varphi_{nl}. \quad (26)$$

We have calculated $M_{cog}(nl)$ in two cases: in single-channel approximation, when $M_{cog}(2P) = 3954$ MeV was obtained, and also taking into account creation of virtual loops $q\bar{q}$, which are important for the states above the open charm threshold and give rise to flattening of confining potential [28]; in the last case $M_{cog}(3P) = 4295$ MeV, $M_{cog}(2P) = 3943$ MeV (which is by 9 MeV smaller than without flattening effect), and then due to fine structure (FS) splittings the mass $M(2^3P_1) = 3934$ MeV is calculated.

For a multiplet nP a spin-orbit $a_{so}(nP)$ and tensor $t(nP)$ splittings are calculated here taking spin-orbit and tensor potentials as for one-gluon-exchange interaction, although as shown in [29], second order ($\alpha_{fs}^2(\mu)$) corrections appear to be not small for the $1P$ multiplet; their contribution can reach $\lesssim 30\%$.

$$a_{so}(nP) = \frac{1}{2\omega_c^2} \left\{ \frac{4}{3} \alpha_{fs} \langle r^{-3} \rangle_{nP} - \sigma \langle r^{-1} \rangle_{nP} \right\} + t(nP), \quad (27)$$

$$t(nP) = \frac{4}{3} \frac{\alpha_{fs}}{\omega_c^2} \langle r^{-3} \rangle_{nP}. \quad (28)$$

We take here $\alpha_{fs} = 0.37$, which provides precise description of the fine-structure (FS) splittings for the 1^3P_1 charmonium multiplet, if second order corrections are taken into account [29]. For the $2P$ multiplet the masses: $M(2^3P_2) = 3963$ MeV, $M(2^3P_1) = 3934$ MeV, $M(2^3P_0) = 3885$ MeV, $M(2^1P_1) = 3943$ MeV are obtained. Notice, that one cannot exclude that for the states above open charm threshold the FS splittings may be smaller or totally screened due to coupling to the DD^* channel, and even the order of the states with different J may be changed.

Below we use the following mass differences:

$$\begin{aligned} M_{cog}(2P) - M_{cog}(1P) &= 425 \text{ MeV}, \quad M_{cog}(3P) - M_{cog}(1P) = 770 \text{ MeV}, \\ M_{cog}(3P) - M_{cog}(2P) &= 350 \text{ MeV}, \end{aligned} \quad (29)$$

In Table 2 the m.e. \mathcal{E}_{if} (in GeV) between n^3P_1 ($n = 1, 2$) and m^3S_1 states are given; in some cases, if the value of \mathcal{E}_{if} is small and results strongly depend on $\alpha_{fs}(\mu)$ used, we give two variants: first, with "normal" FS splittings and in second case FS effects are totally suppressed.

To control an accuracy of our calculations in Table 2 we give also the partial widths of the dipole transitions: $1^3P_1 \rightarrow J/\psi\gamma$, $2^3S_1 \rightarrow \chi_{c1}(3510)$, and $1^3D_1 \rightarrow \chi_{c1}(3510)$, and in

TABLE II: E1 transition rates. The m.e. $\langle X(3872)|r|n^3S_1\rangle$ ($n = 1, 2$) includes admixture from the 1^3P_1 component with $c_1 = 0.153$; experimental data from [30]-[32].

Transition	E_γ	S_{if}	\mathcal{E}_{if}	$\Gamma(i \rightarrow f)$ (keV)		
$i \xrightarrow{E1} f$	(MeV)		(GeV) $^{-1}$	this paper	BG[13]	experiment
$1^3P_1(3510) \quad 1^3S_1(3097)$	389	$\frac{1}{9}$	1.927	315		317 ± 25 [30]
$2^3P_1(3872) \quad 1^3S_1(3097)$	697	$\frac{1}{9}$	$0.104^a)$ $0.216^b)$	5.3 22.8	11.0	
$X(3872) \quad 1^3S_1(3097)$	697	$\frac{1}{9}$	$0.396^c)$	76.6		
			$> 0.292^d)$	> 41.7		
$2^3S_1(3686) \quad 1^3P_1(3510)$	171	$\frac{1}{9}$	-2.104	31.9		30.6 ± 2.2 [31]
$2^3P_1(3872) \quad 2^3S_1(3686)$	181.5	$\frac{1}{9}$	3.02	78.6	63.9	
$X(3872) \quad 2^3S_1(3686)$	181.5	$\frac{1}{9}$	2.70	62.8		
$1^3D_1(3770) \quad 1^3P_1(3510)$	252.9	$\frac{1}{18}$	2.767	89	199	70 ± 17 [32] 80 ± 24 [27]
$2^3P_2(3872) \quad 1^3D_1(3770)$	97.7	$\frac{1}{18}$	-2.776	5.2	3.7	
$X(3872) \quad 1^3D_1(3770)$	97.7	$\frac{1}{18}$	-2.32	3.6		

^{a)} $\alpha_{fs} = 0.37$ in spin-orbit potential.

^{b)} FS interaction is totally suppressed.

^{c)} Both admixture of the 1^3P_1 state and FS splittings with $\alpha_{fs} = 0.37$ are taken into account.

^{d)} The lower limit refers to the case when FS interaction is totally suppressed.

all three decays good agreement with existing experimental data [30]-[32] is obtained (here the 2^3S_1 and 1^3D_1 states are identified with ψ' and ψ''). In our calculations the $2S - 1D$ mixing is not taken into account.

From Table 2 one can see that for $X(3872)$ as a purely 2^3P_1 state, the transition rate $\Gamma_1 \equiv \Gamma(2^3P_1 \rightarrow J/\psi\gamma)$ strongly depends on FS potential used. For this transition the m.e. $\mathcal{E}_{21} = 0.10 \text{ GeV}^{-1}$ is small, if in spin-orbit potential “normal” $\alpha_{fs} = 0.37$ is used, and $\Gamma_1 = 5.3 \text{ keV}$ is also small, being less than in [13], where a larger $\mathcal{E}_{21} = 0.15 \text{ GeV}^{-1}$ and $\Gamma_1 = 11 \text{ keV}$ were calculated.

In the case when FS interaction is neglected, or suppressed, then m.e. $\mathcal{E}_{21} = 0.216 \text{ GeV}^{-1}$

is larger (in [13] $\mathcal{E}_{21} \sim 0.276 \text{ GeV}^{-1}$), and the partial width $\Gamma_1 = 22.8 \text{ keV}$ is 4 times larger. In both cases discussed a contribution from the 1^3P_1 component was also neglected and the ratio R is large, $R > 4$. Notice, that in [33], using an analogy with the radiative decays of $\chi_{bJ}(2P)$ in bottomonium, a smaller value of this ratio, $R = 1.64 \pm 0.25$, was predicted.

However, if admixture from the 1^3P_1 state, as in (21), is taken into account, then the transition rate Γ_1 increases, independently of a strength of FS interaction used. With $c_1 = 0.153$ and suppressed FS interaction we obtain the lower limit for Γ_1 ,

$$\Gamma_1(X(3872) \rightarrow J/\psi\gamma) \geq 41.7 \text{ keV}. \quad (30)$$

This transition rate reaches a larger value, $\Gamma = 76.6 \text{ keV}$, if in FS potential the same $\alpha_{fs} = 0.37$, as for the $1P$ states, is used. On the contrary, the partial width $\Gamma_2 \equiv \Gamma(X(3872) \rightarrow \psi'\gamma)$ decreases (by 20%), owing to admixture c_1 in the w.f. of $X(3872)$ and negative m.e. $\langle 2^3S_1 | r | 1^3P_1 \rangle$. As a whole, the ratio R is becoming smaller and can change in wide range:

$$0.53 \leq R \leq (0.8 \pm 0.2) \text{ (th)}, \quad (31)$$

where the upper limit refers to the case when FS interaction is strong and theoretical error comes from a variation of α_{fs} , while the lower limit refers to the case when FS interaction is suppressed.

These values of R are in agreement with the Belle measurements of the $X(3872)$ radiative decays where a restriction $R < 2.1$ was observed [16]. At the same time our limit, $R \leq (0.8 \pm 0.2)$, does not agree with $R = 3.4 \pm 1.4$ from the BaBar experiment, being also much smaller than in many-channel calculations [13],[14], where the ratio $R \simeq 5$ was obtained.

Thus we conclude that precise measurements of R are of great importance for understanding the nature of $X(3872)$: firstly, for definition of admixture of the 1^3P_1 component in the w.f. of $X(3872)$ and secondly, for understanding of FS effects in higher resonances, which lie above open-charm threshold.

Notice, that the partial width $\Gamma_1(2^3P_1(3872) \rightarrow J/\psi\gamma)$ is not very small even for a pure 2^3P_1 state, if a contribution from spin-orbit interaction is suppressed, and the partial width increases 4 times:

$$\Gamma_1^{(0)}(2^3P_1(3872) \rightarrow J/\psi\gamma) = 22.8 \text{ keV}, \quad (32)$$

while in this case $\Gamma_2^{(0)}(2^3P_1(3872) \rightarrow \psi'\gamma) = 85.5 \text{ keV}$ increases only by $\sim 8\%$; however, their ratio remains large,

$$R_0(2^3P_1) = 3.75. \quad (33)$$

The situation changes, if FS interaction is suppressed, but the w.f. of $X(3872)$ contains admixture of the 1^3P_1 state (21). Then the m.e. $\langle X(3872)|r|J/\psi\gamma\rangle = 0.512 \text{ GeV}^{-1}$ and $\Gamma_1 = 128 \text{ keV}$ reaches the maximum value, but the m.e. $\langle X(3872)|r|\psi'\gamma\rangle = 2.80 \text{ GeV}$ changes by only $\sim 10\%$ and $\Gamma_2 = 67.6 \text{ keV}$; so that their ratio has a minimal value:

$$R_{min}) = 0.53. \quad (34)$$

To define FS splittings of the nP multiplets we use following m.e.

$$\begin{aligned} \langle 2P|r^{-1}|1P\rangle &= 0.134 \text{ GeV}, & \langle 2P|r^{-3}|1P\rangle &= 0.123 \text{ GeV}^3, \\ \langle 3P|r^{-1}|1P\rangle &= 0.080 \text{ GeV}, & \langle 3P|r^{-3}|1P\rangle &= 0.112 \text{ GeV}^3, \\ \langle 3P|r^{-1}|2P\rangle &= 0.133 \text{ GeV}, & \langle 3P|r^{-3}|2P\rangle &= 0.134 \text{ GeV}^3. \end{aligned} \quad (35)$$

Then for $\alpha_{fs} = 0.37$ one finds

$$\langle X(3872)|r|J/\psi\gamma\rangle = 0.104 \text{ GeV}^{-1}, \quad (36)$$

which is two times smaller than in the spin-average case, when $\langle X(3872)|r|J/\psi\gamma\rangle = 0.216 \text{ GeV}^{-1}$, and the partial width is large,

$$\Gamma_1(X(3872) \rightarrow J/\psi\gamma) = 76.6 \text{ keV}. \quad (37)$$

The m.e. $\langle X(3872)|r|\psi'\gamma\rangle = 2.70 \text{ GeV}^{-1}$ in this case, giving

$$\Gamma_2(X(3872) \rightarrow \psi'\gamma) = 62.8 \text{ keV}, \quad (38)$$

and the ratio $R = 0.82$. However, if a stronger $\alpha_{fs} \geq 0.45$ is used, this ratio may reach a larger value, ~ 1.0 , so that R can vary in the range,

$$R = 0.8 \pm 0.2 \text{ (th)}. \quad (39)$$

Thus, while both spin-orbit splitting and admixture c_1 from the 1^3P_1 state are present, then the partial widths Γ_2 and Γ_1 turn out to be of the same order.

In our calculations above we have disregarded the contribution of the γ emission from the DD^* intermediate state in the radiative decays of $X(3872)$ into J/ψ or ψ' . To estimate this part of the γ emission we refer to calculations done in [34]. It was found there that the channel DD^* contributes to the $J/\psi\gamma$ final state less than 3.6 keV and to the $\psi'\gamma$ final state less than 0.01 keV, i.e. these contributions are smaller as compared to changes in corresponding partial widths due to variations of the coupling α_{fs} in the range $0.25 - 0.45$ (see Table 2). Therefore in this paper we have disregarded possible effect of γ emission from DD^* intermediate states.

IV. CONCLUSIONS

We study the exotic charmonium state $X(3872)$ with $J^{PC} = 1^{++}$ in the CC approach, where a coupling to the DD^* channels is determined by the parameter-free string-breaking mechanism. Due to this coupling the 2^3P_1 charmonium state is shifted down to the D^0D^* threshold and its w.f. acquires admixture from the 1^3P_1 $c\bar{c}$ state. Such mixing of the 2^3P_1 and 1^3P_1 states is not large, corresponding to the mixing angle $\theta = 8.8^\circ$.

Owing to this admixture the transition rate $\Gamma_1(X(3872) \rightarrow J/\psi\gamma)$ increases several times and reaches the value in the range $45 - 80$ keV. At the same time the transition rate $\Gamma_2(X(3872) \rightarrow \psi'\gamma)$ decreases by $\sim 15\%$. As a result their ratio has following features:

1. The ratio $R = 0.53$, if spin-orbit interaction is totally suppressed.
2. The ratio $R = 0.82$, if spin-orbit interaction is defined by the FS coupling, $\alpha_{fs} \sim 0.37$ and can reach the larger value ~ 1.1 for a larger α_{fs} .

The partial width of $X(3872) \rightarrow \psi''\gamma$ appears to be small, ~ 4 keV.

Our calculations support the Belle result that $R(\text{exp.}) < 2.1$, while a larger number, $R = 3.4 \pm 1.4$ puts an additional restrictions on the value of admixture c_1 in the w.f. of $X(3872)$.

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